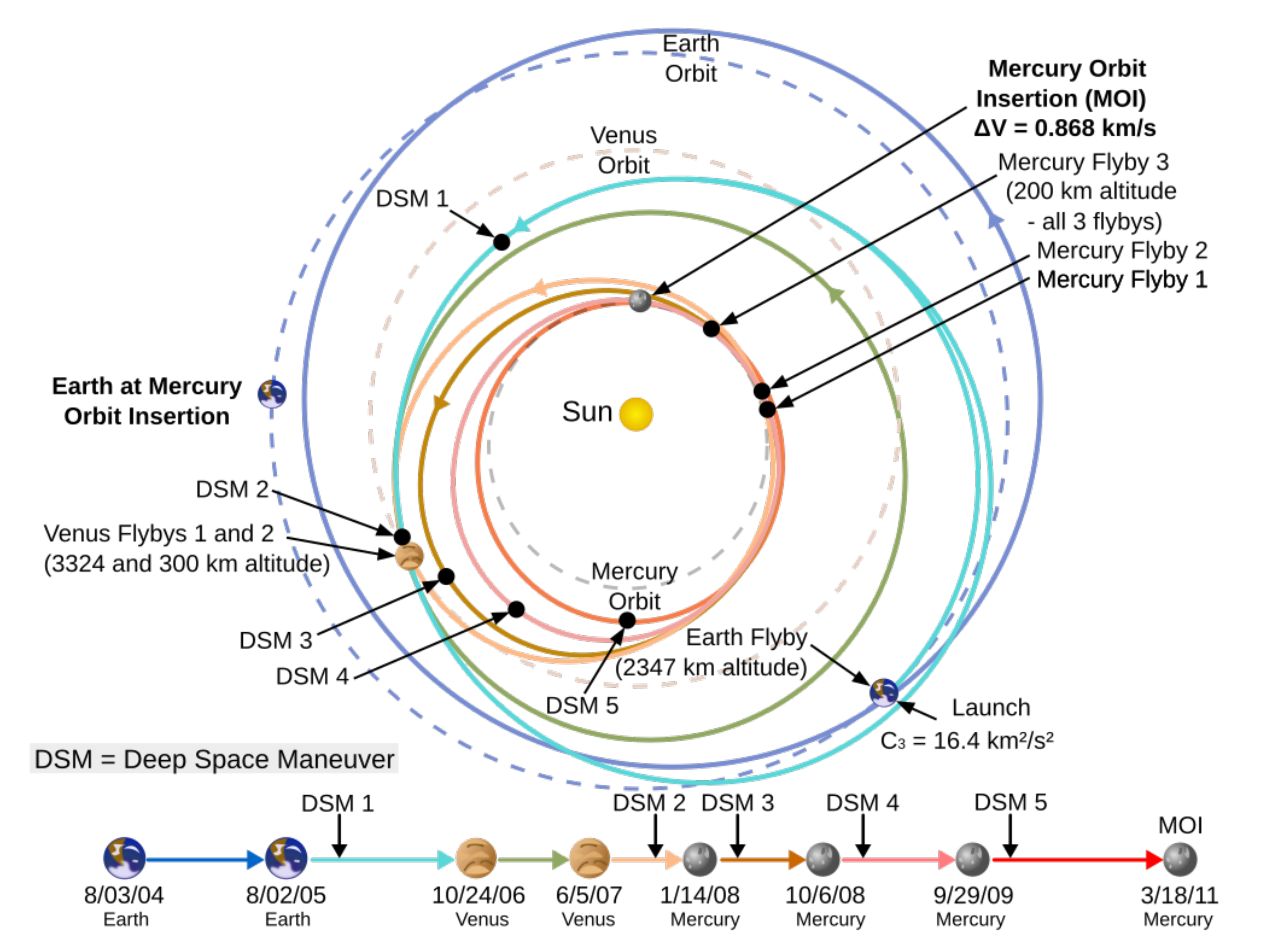
News and Reminders

Homework 1 is due now.

Next reading quiz: this Wednesday, 9/11.

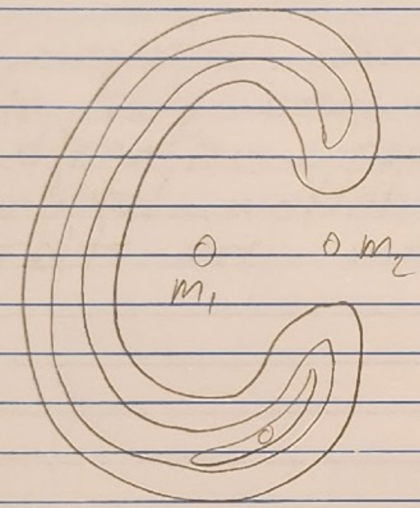
JC 1: Monday, 9/16 (in one week) - *Evidence for Hidden Nearby Companions to Hot Jupiters* - Sarah Stamer

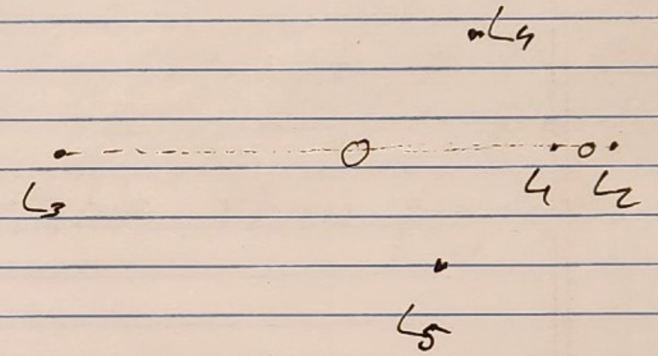
Messenger Hohmann Transfer Orbits



Jacobi integral relates third particle's position and velocity at any point. It is the total energy of the third particle rel. to rotating ref. frame. Not fully solved, but determines forbidden regions.

horseshoe orbits





L_1, L_2 and L_3 are saddle points of the total potential $\Rightarrow$ they are unstable.

L_4 and L_5 together form a 0-velocity curve with smallest C_J. A small perturbation to a particle here causes it to librate around the point.

~~L_1, L_2 and L_3~~ horseshoe/tadpole slide

Particles orbiting L_4 or L_5 are in tadpole orbits.

<u>Hill sphere:</u>

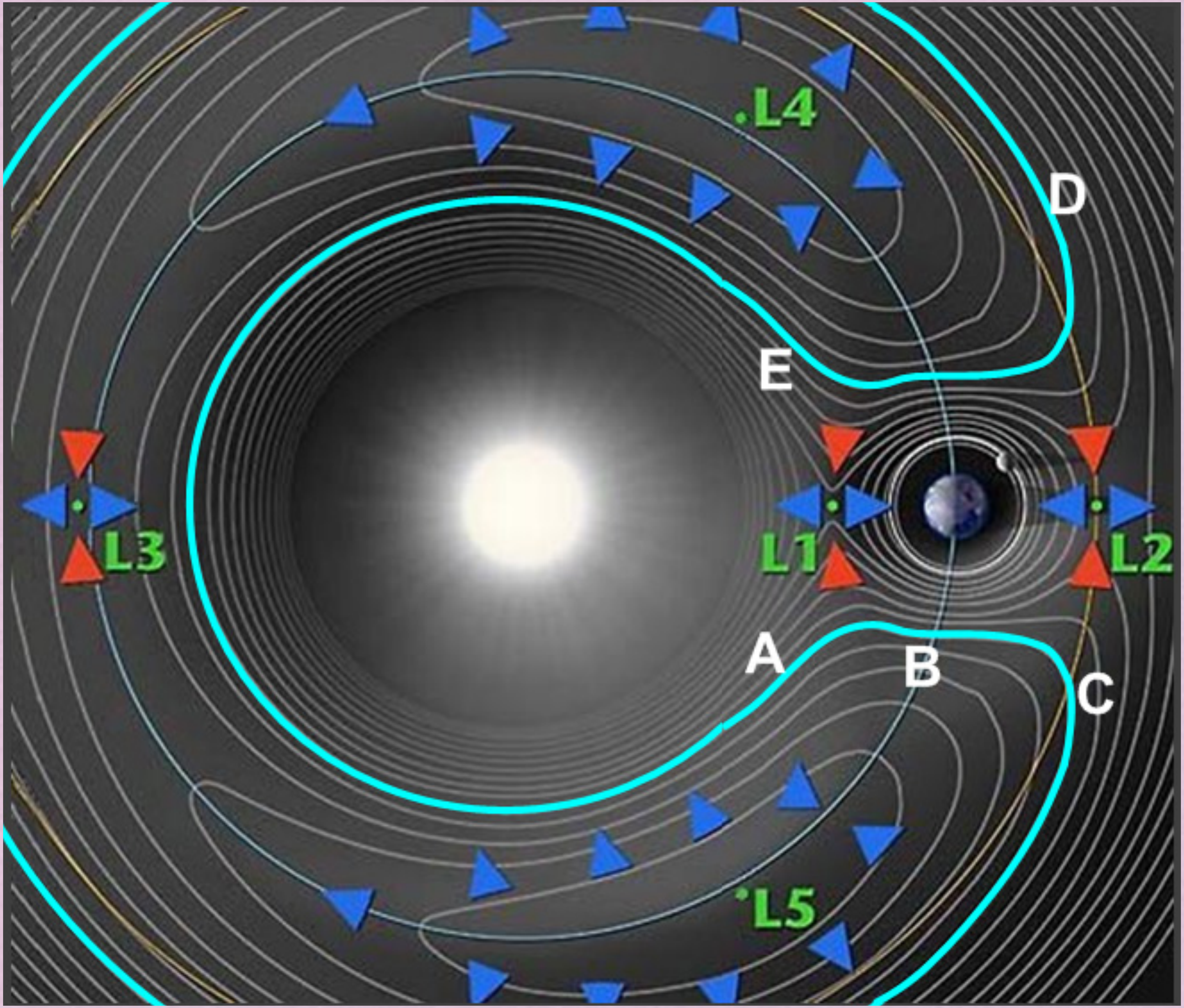
slide

Limit to a $\overbrace{\text{secondary's}}^{m_2}$ gravitational dominance.

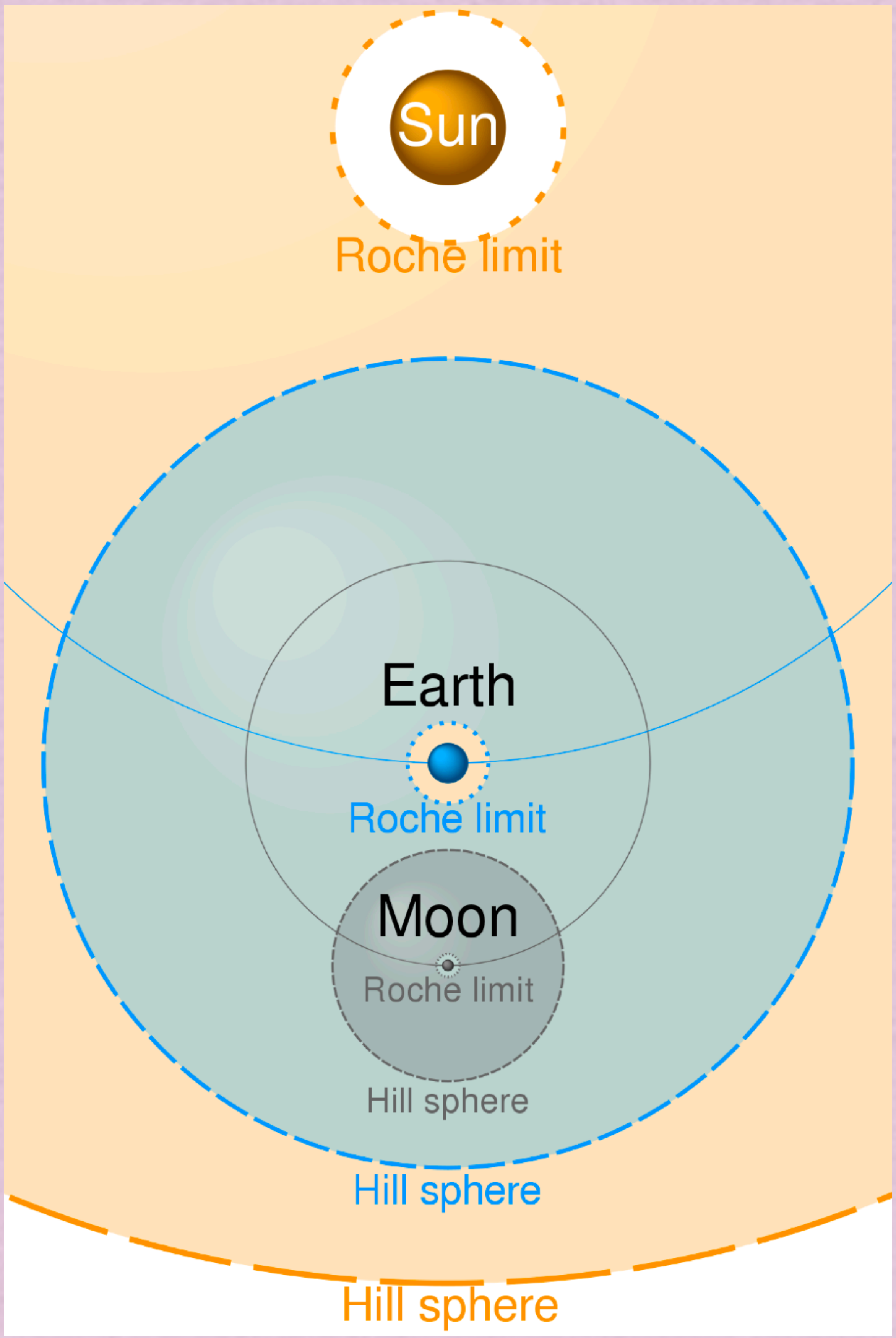
$$R_H = \left(\frac{m_2}{3(m_1 + m_2)} \right)^{1/3} a$$

reaches to L_1 and L_2

Lagrange points



Hill Sphere


Sun
Roche limit
Earth
Roche limit
Moon
Roche limit
Hill sphere
Hill sphere
Hill sphere

Satellites are well within their planet's Hill sphere, for stability.

Objects need to be outside planet's HS to be in stable heliocentric orbits.

Computer simulations show that retrograde (in this case, orbiting the planet in the direction opposite that of the planet's orbital motion around the Sun) satellite orbits are more stable than prograde ones.

Regular vs. Chaotic Motion:

We may not be able to analytically solve for motion of orbiting objects most of the time but, at least we can hope to do so numerically.

This is true for regular motion; that can be well described numerically for an arbitrary-long finite time.

So, generally $\delta(t) = \delta(0) + ct$

distance between two particles at time t

initial distance between two particles

But, if motion depends too sensitively on the initial conditions, then its behavior becomes effectively unpredictable, and chaos ensues.

For chaotic motion: $\delta(t) \sim \delta(0) e^{\gamma_c t}$

$\gamma_c \Rightarrow$ Lyapunov exponent / $\gamma_c^{-1} \Rightarrow$ Lyapunov time

Class Q: how many Lyapunov times for a 10^{-10} perturbation to result in a 100% discrepancy? Round to nearest integer

Interestingly, in the Solar System chaotic behavior develops over timescales orders of magnitude $> \gamma_c^{-1}$

Often regions of chaos are associated with resonances ratios of frequencies approximated by rational numbers.

Chaos

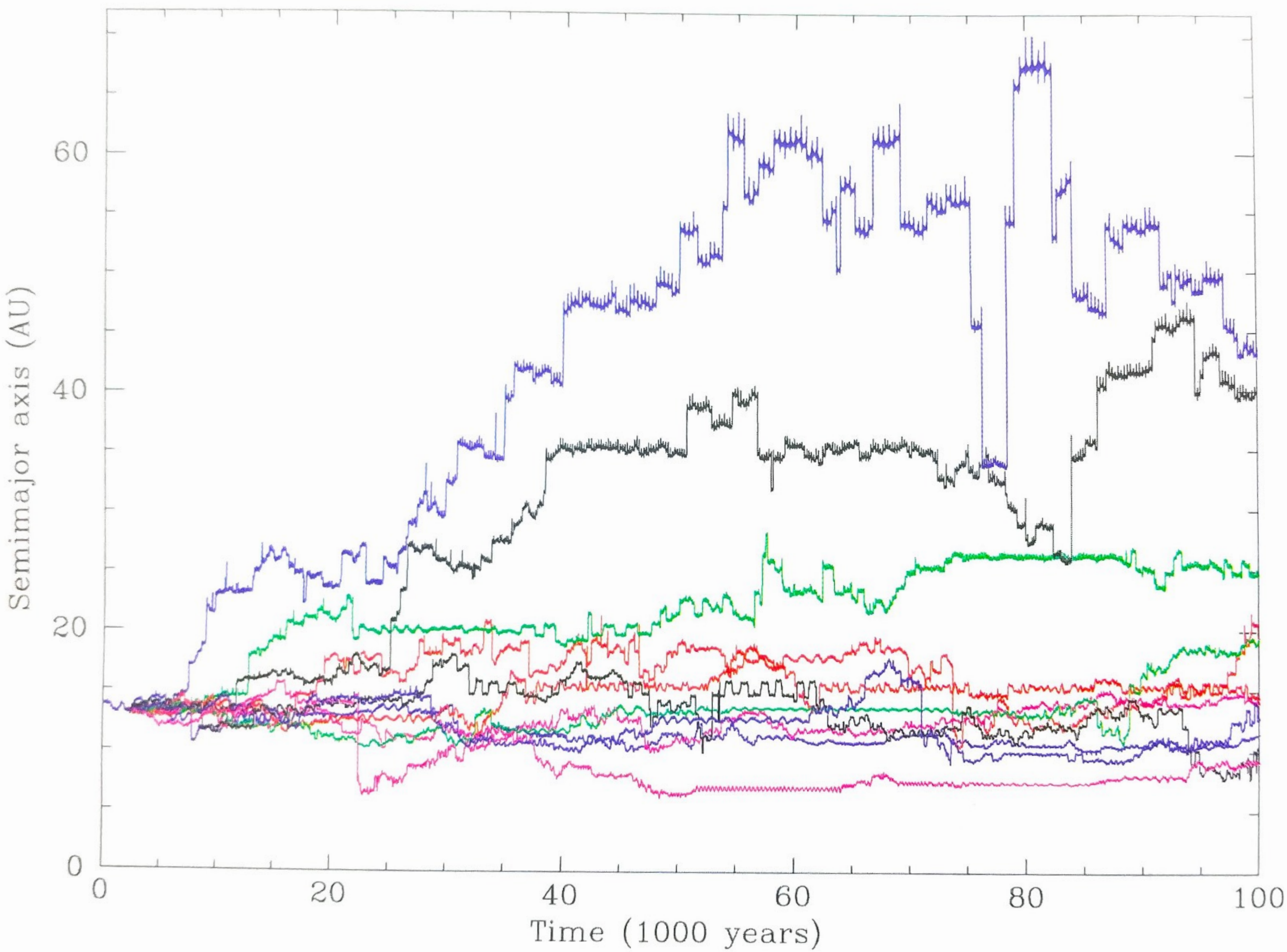


Figure 2.11

Mean motion resonance: (MMR)

Most long-term perturbations to the motion of a Solar System body are periodic.

Sometimes, perturbations (even small ones) add up coherently $\Rightarrow$ large-amplitude, long-period response

Consider the equation of motion of 1D forced harmonic oscillator:

$$\frac{m\,d^2x}{dt^2} + m\omega_0^2 x = F_f \cos(\omega_f t)$$

ω_0: natural frequency
ω_f: forcing "

integrate to get equation for x:

$$x = \frac{F_f}{\underbrace{m(\omega_0^2-\omega_f^2)}_{\text{usually small, so } x \text{ has low amplitude}}} \cos(\omega_f t) + C_1 \cos(\omega_0 t) + C_2 \sin(\omega_0 t)$$

C_1 and C_2 are constants corresponding to initial conditions.

What happens when $\omega_f \approx \omega_0$?
A large-amplitude response occurs even if F_f is small.

When $\omega_0 = \omega_f$, solution is:

$$x = \frac{F_f}{2m\omega_0} t \sin(\omega_0 t) + C_1 \cos(\omega_0 t) + C_2 \, \cancel{\cos}\,\sin(\omega_0 t)$$

steady (secular) growth!

Mean motion resonance (MMR):

resonance applied to orbital period

e.g. $P_{ratio} = \frac{N}{N+1}$ or $\frac{N}{N+2}$ or ~~~

higher $N:(N\pm1)$ resonances are stronger because closer to secondary (relative to primary)

Class: why? ... but also less stable

Resonances in Asteroid belt slides

Class: at what resonance with Jupiter do the Trojans orbit?

Stability of the Solar System:

N-body problem with fairly large N.

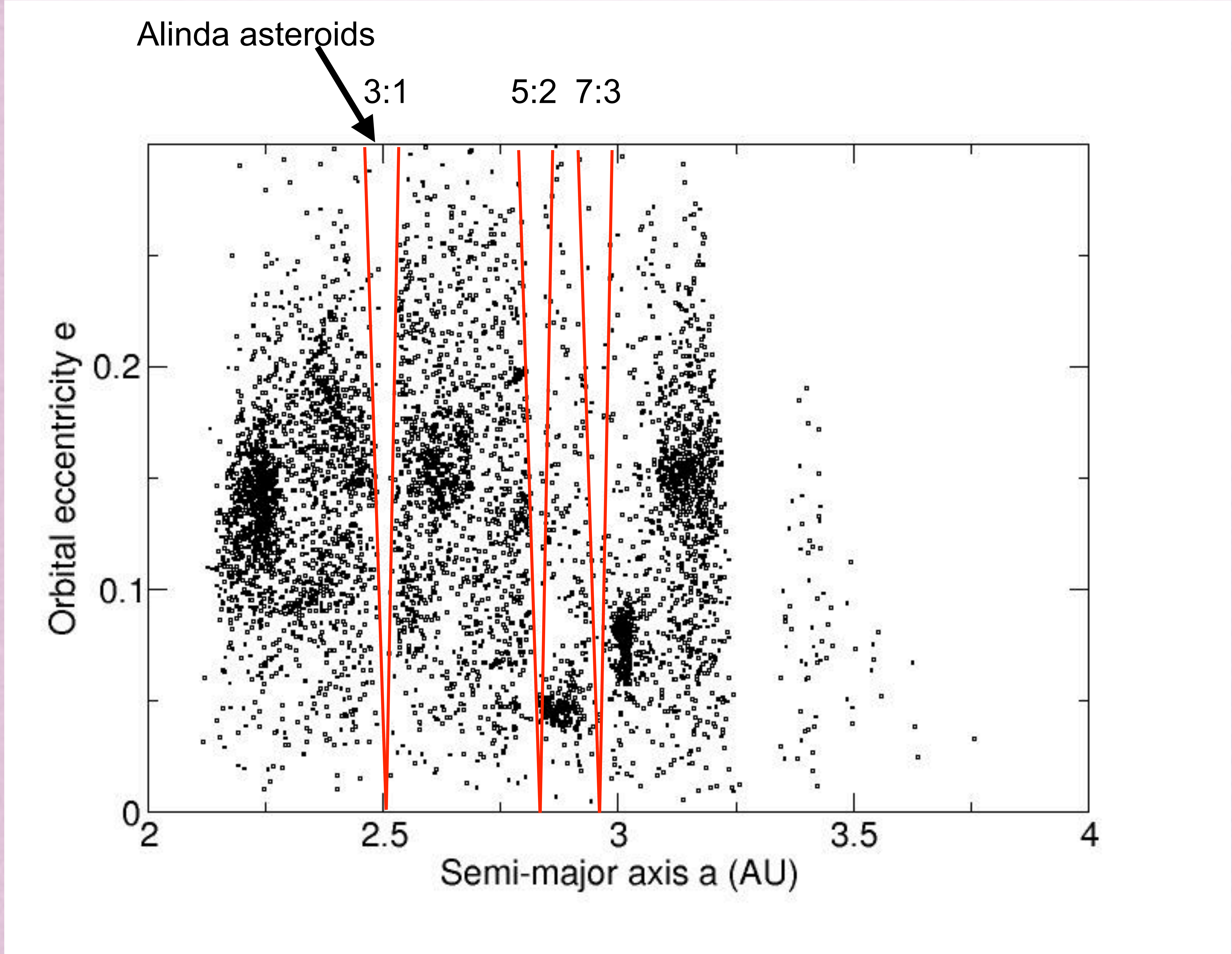
Lyapunov time: 2–230 Myr
$\Rightarrow$ planets orbits are chaotic over long timescales

slide

This is because orbital resonances appear on a wide range of timescales. Could be:
- orbital resonance
- secular resonance slide
- spin-orbit resonance

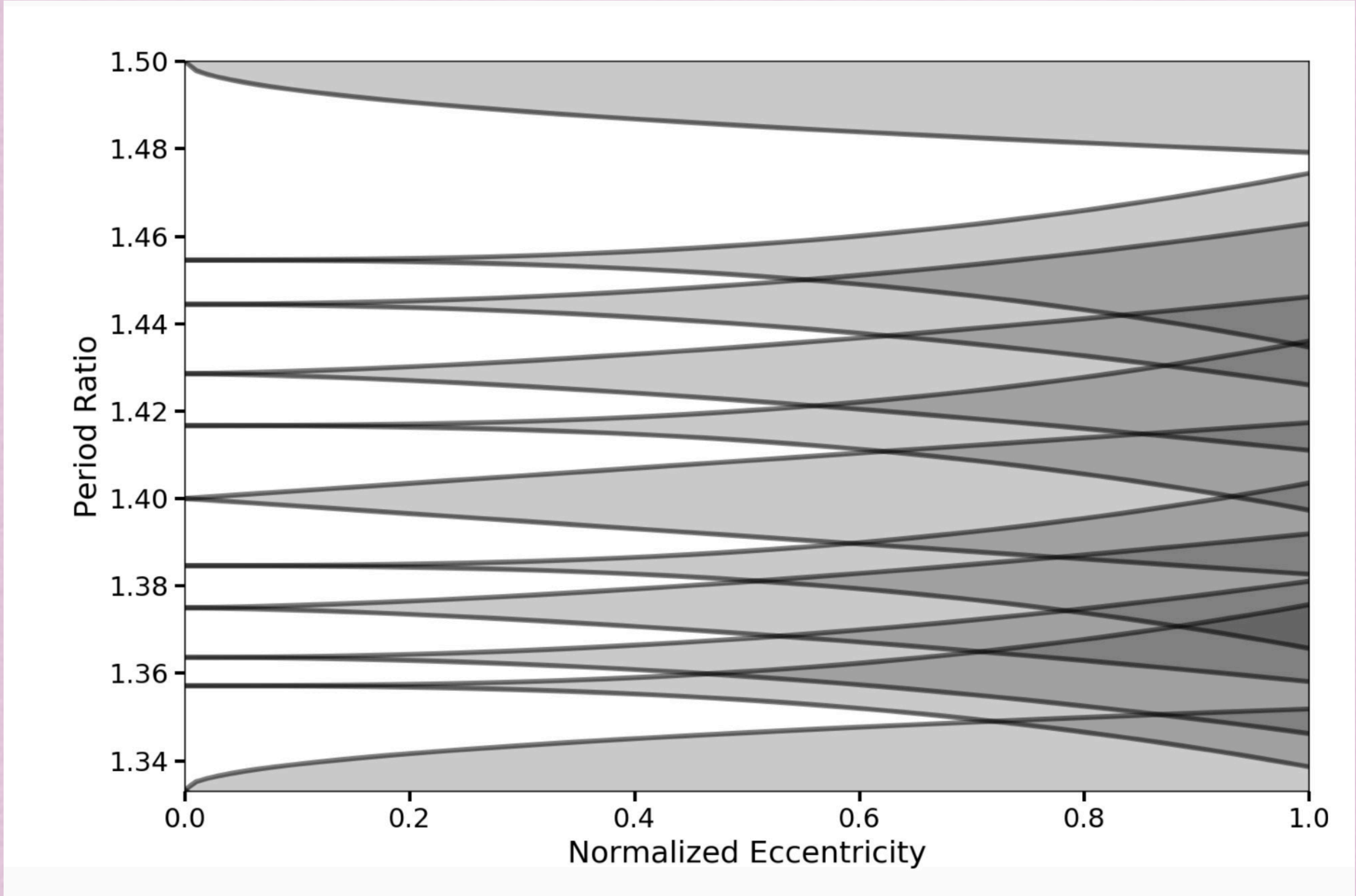
slide on future Mercury and other planets' orbits

Resonances

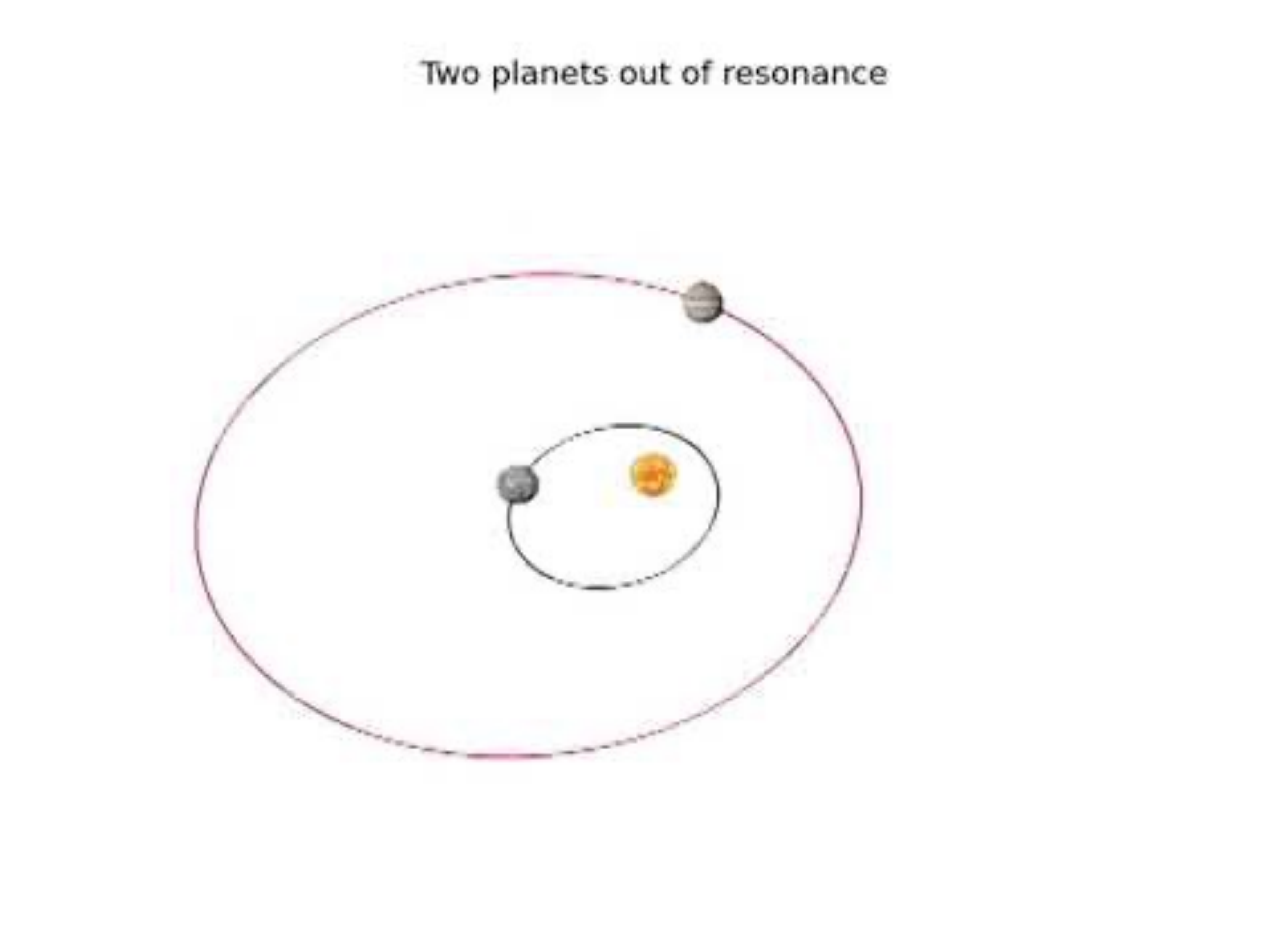


Alinda asteroids
3:1
5:2
7:3
Orbital eccentricity e
0.2
0.1
0
2
2.5
3
3.5
4
Semi-major axis a (AU)

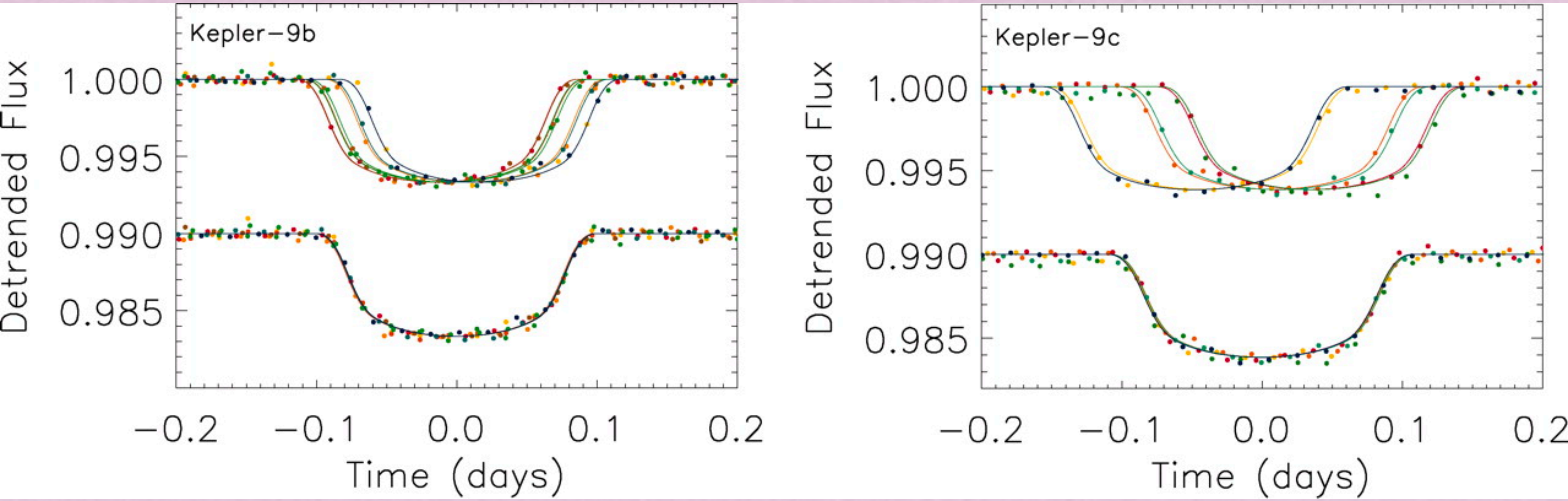
Mean Motion Resonance vs. Eccentricity



Secular Resonance



Transit Timing Variations



Holman et al. (2010)

- For Kepler systems, it is often the only way to measure planet masses