Bound and Unbound Orbits:

slide

centripetal force: to keep object in circ. orbit

n = mean motion OR avg. ang. speed

$$\vec{F}_c = \mu n^2 \vec{r}$$

$$= \mu \left(\frac{2\pi}{P}\right)^2 \vec{r} = \mu \frac{4\pi^2}{4\pi^2} \frac{V_c^2}{r^2} \vec{r}$$

$$= \mu \frac{V_c^2}{r} \hat{r}$$

K.E. = P.E.

$$\mu \frac{V_c^2}{r} \hat{r} = \frac{G m_1 m_2}{r^2} \hat{r}$$

$$V_c = \sqrt{\frac{G}{r}(m_1 + m_2)}$$

$$= \sqrt{\frac{GM}{r}}$$

$$E = \frac{1}{2}\mu v_c^2 - \frac{GM\mu}{r} = \cancel{GM}$$

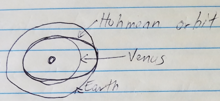
$E > 0 \Rightarrow$ K.E. > P.E. $\Rightarrow$ unbound
hyperbola!

($E < 0 \Rightarrow$)

$E = 0 \Rightarrow$ parabola (unstable)

Die Q: example of hyperbolic orbit

Space craft in the Solar System and Hohmann orbits



elliptical, so..

gravity asits used for altering orbit ~~and~~ (slow or speed up) and save fuel when sending spacecraft beyond Venus or Mars

use vis viva eqn. to calculate velocity needed to leave Earth or arrive @ target

from E eqn: $v^2 = GM\left(\frac{2}{r} - \frac{1}{a}\right)$

The three-body problem:

Most of the time there are more than two bodies in a system,

with

⇒ but even just three bodies, there are not enough integrals of motion to completely specify the motion of each body (6N) ⟺ 3 posit, 3 vel, OR 6 orbital elements OR ...

class

10 {
initial position ×3 (one for each dimension)
linear velocity ×3 (" " " ")
angular momentum ×3 (" " " ")
total energy of the system
}

Analytic solutions are only possible in some limiting cases.
Otherwise numerical integrations are needed.

Circular restricted three-body problem:

third body has negligible mass, circular orbit

Earth + Moon + spacecraft
Sun + Jupiter + asteroids
Binary star system + gas particle

3x Q: give me an ex. of circular restricted 3-body prob.

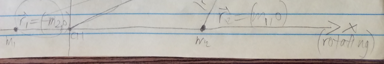
Elliptic restricted 3-body problem is the subject of (considerable) research activity.

The problem is thus reduced to the study of the motion of only the test particle in the field of the two co-orbiting primaries (because the orbits of m_1 and m_2 conform to the solution of the 2-body problem and are thus Known.

Recall: with judicious coordinate change $\left(\vec{r} = \vec{r} - \vec{r}_2\right)$, the symmetry of the 2-body problem can be used to find an equivalent 1-body problem.
$\Rightarrow$ so we go from ~~18~~ 12 dimensions to 6
$6 < 10$ so we can solve!

Need to move to rotating coordinate system w/ origin @ CM of the two primaries, w/ x-axis passing through them and z-axis perp. to orbital plane

$G = 1$
$m_1 + m_2 = 1$
$X_1 + X_2 = 1$



Jacobi constant and Jacobi integral

Total energy is not conserved in the circular restricted 3-body problem (because grav. effect of test particle on the primaries is neglected).
The potential energy depends on time

$$\frac{1}{2}\frac{d}{dt}\left(v^2\right) = -\frac{dU}{dt}$$

$$\frac{1}{2}v^2 + C_1 = -U + C_2$$

$$\frac{1}{2}v^2 + U = C$$

$$\frac{1}{2}v^2 - \frac{1}{2}\left(x^2+y^2\right) - \frac{m_2}{\vec{r}-\vec{r}_1} - \frac{m_1}{\vec{r}-\vec{r}_2} = C$$

multiply by -2 (convention):

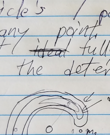
$$\boxed{x^2+y^2 + \frac{2m_2}{|\vec{r}-\vec{r}_1|} + \frac{2m_1}{|\vec{r}-\vec{r}_2|} - v^2 = C_J}$$

Jacobi constant/integral related third particle's position and velocity at any point.

Not ~~ideal~~ fully solved

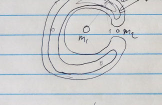
But Does the determination of forbidden regions

diff. Jacobi csts,



unstable

L_1, L_2, L_3 are saddle points of the total potential



$y=0$

L_4, L_5 together form 0-vel. curve with smallest C

if small perturbation, they librate around L_4 or L_5 (don't go past secondary)