PHYS 480/581 Homework #1

Fall 2024 Due Monday, Sep 9 in class

1) **The coplanar Three-Body-Problem.** (*Mandatory for PHYS 581 students; optional for PHYS 480 students, who may choose to do it for bonus points.*)

Consider a coplanar planetary system with a massive mass m1 at its center and two planets (masses m2 and m3), as in Figure 1. The two bodies initially orbit the central mass on circular orbits, and start in opposition ($\delta\varphi = \pi$). The mutual gravitational perturbations will modify the eccentricities (e) and semi-major axes (a), and therefore the shape of the orbits. Large changes should occur when the two planets are close in conjunction. The system is called stable if close encounters are excluded at all times. We will examine the stability of the system as a function of the masses and the initial conditions, shown in Figure 1. For small initial separations Δ of the two masses m2 and m3, we expect larger gravitational perturbations of the two masses and therefore higher changes in e and a.

Assuming m2 and m3 are small and initially on circular orbits, we have the following criterion for stability for the initial separation of the two circular planetary orbits (where μ corresponds to the reduced mass)

$$\Delta_c \simeq 2.40\,(\mu_2 + \mu_3)^{1/3}.$$

In order to study the stability, we start with a given Δ and perform the integration of the trajectory for 1000 orbits. Instability can be spotted by searching for close encounters. A close encounter occurs when the distance between the two planets is smaller than the Hill radius of the more massive of the two bodies. The Hill radius is given by

$$R_{H} \approx a\sqrt[3]{\mu/3},$$

where a is the distance of the planet to the star and μ corresponds to the mass ratio of the smaller object to the more massive one. In the exercise, we determine numerically the critical distance Δc . For Δ values lower than Δc , the system is unstable. For Δ values above, the system is stable. The precise value of Δc might depend on the applied numerical integrator. REBOUND can automatically find collisions between the objects if they were given a radius when added to the simulation (https://rebound.readthedocs.io/en/latest/collisions/). Start by defining a function which will be called for each collision. You will use this feature to add a global counter which we increase if the planets get closer than their Hill's sphere.

Use the standard IAS15 integrator and (a) masses m1 = 1; m2 = m3 = 10^{-5} ; a2 = 1; a3 = a2 + Δ; e2,3 = 0. Add the masses to the REBOUND simulation and indicate their collision radius by using the argument r in the function sim.add. Simulate the system for approximately 10,000 orbits (one period is approximately 2π), count the numbers of close encounters and plot the evolution of the semi-major axes and eccentricities of the two planets for five different values of the initial separation Δ: 10%, 50%, 100%, 150% and 1000% of Δc . (b) Repeat exercise (a) with masses m1 = 1, m2 = 10−5 , m3 = 10−7 , a2 = 1, a3 = a2 + Δ, e2,3 = 0. Describe your findings, and compare the evolution of the orbital elements (semi-major axes and eccentricities) to what was seen in the case of the two-body problem exercise we did in class.

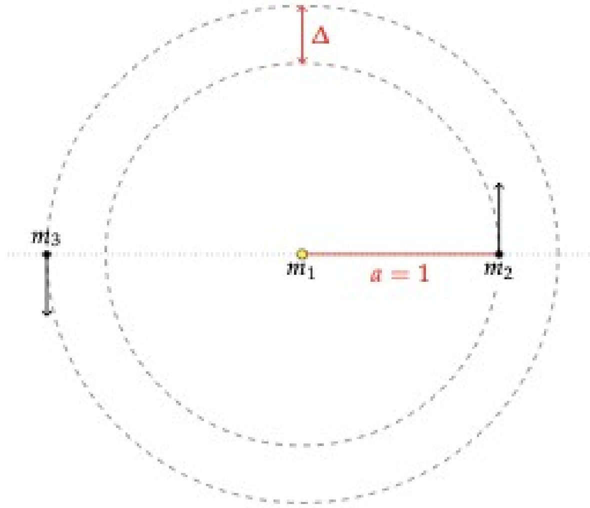

Figure 1: Schematic representation of a three body problem, where two smaller test masses m_2 and m_3 orbit a larger central mass m_1, it holds $m_2 + m_3 \ll m_1$. The initial semi-major axis of mass m_2 is normed to 1 in respect to the central mass m_1, and the semi-major axis of m_3 to $1 + \Delta$. The initial locations of the smaller bodies are opposed, $\delta\phi = \pi$, (after Gladman, 1993).

2) **Radial Velocity.** An astronomer has been collecting radial velocity observations of a nearby star with a mass of 0.6 M_{Sun}. She reports a signal with a semi-amplitude K = 1.6 m/s and a period of 11 days. The orbit appears to be circular, but the inclination of the planet is unknown. The relevant equation(s) are in ch. 12 of your textbook.

a) What is the minimum mass of the planet?
b) Assuming that inclination angles are randomly distributed, what is the most likely mass of the planet? How does this compare to the answer you found in part (a)?

3) **Kepler's Laws vs. Newton's Laws.** Newton's laws can be used to obtain a revised version of Kepler's Laws of Motion. Use Newton's laws of motion and gravity to revise Kepler's Third Law to:

$$P^2 = \frac{4\pi^2 a^3}{G(m_1 + m_2)}$$

4) Problem 2.4 (a to e) from the **second** edition of *Planetary Sciences* (de Pater & Lissauer).

5) **Orbital garbage disposal.** During a space walk outside the international space station, is it safest to throw a piece of garbage vertically down toward the Earth below, up, forward, or backward along the orbit? Why?