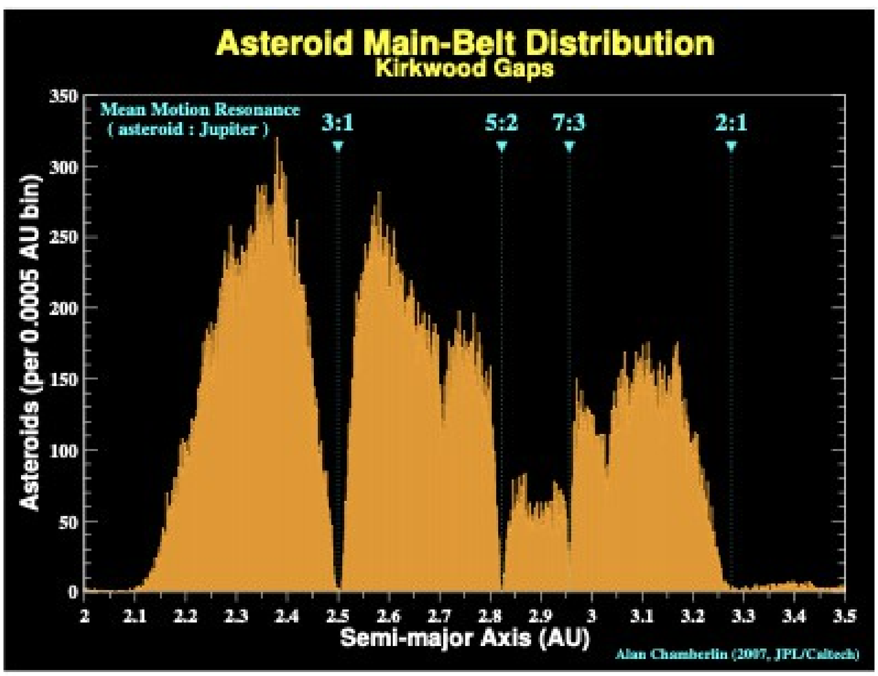
PHYS 480/581 — Homework #2

Fall 2024 — Due Monday, September 30 at class time

1) **Jupiter and Kirkwood gaps.** (*Mandatory for PHYS 581 students; optional for PHYS 480 students, who may choose to do it for bonus points.*)

In this problem we want to address the gravitational influence of Jupiter on the asteroid belt. Kirkwood gaps correspond to the locations of asteroid orbital resonances with Jupiter. Use inactive particles to represent the asteroids such that the asteroids have no feedback on Sun, Jupiter and Mars. They are just tracer particles. Consider only the Sun, Jupiter and Mars as gravitating objects (so use r->N_active=3). Set up the Sun-Mars-Jupiter system with the help of the NASA Horizons web-interface to get initial conditions for Mars and Jupiter (use Ephemeris Type Vector Table) and randomly add 10 000 inactive particles in the midplane between 2 and 4 AU. Let the system evolve for **several hundreds thousands of years** and look for the Kirkwood gaps (see the figure below). **Note: This will probably require to run the code over night** (depending on your hardware). Generate the following pairs of plots for different times (every 100 000 years, until 1 Myr, to show the formation of the gaps): eccentricity versus semi-major axis; histogram plot of number of asteroids versus semi-major axis. Add the known Kirkwood gaps to your plots. Do you find agreement with your results?



2) **Roche limit.** There is ongoing debate in the planetary science community regarding how Saturn's rings have formed. Could they originally have been a moon that was torn apart by tidal forces? Let's test this theory. Use the following formulation for Roche's limit:
$d = 1.44\ R_p\ (\rho_p/\rho_s)^{1/3}$, where Rp is the radius of the planet, ρ_p is the bulk density of the planet, and ρ_s is the bulk density of the moon. Gather or calculate estimates of Saturn's bulk density and radius. To estimate ρ_s, you can either choose some reasonable moon radius and use the fact that the total mass of the rings is ~ 1.5 x 10 19 kg, or assume the moon was made of the same material as the rings and find what the density of that material is. Calculate the Roche limit of tidal disruption for such a moon. Look up the distance between Saturn and its rings, and decide whether this theory is reasonable.

3) **What's heating Io?** The average outward heat flux due to tidal flexing from the surface of Io is Ftid = 2.25 W/m^2. It is caused by the eccentricity of the satellite's orbit around Jupiter by moons such as Europa and Ganimede, and by the viscous dissipation of tidal flexing of Io's body.

a) Compare that flux with the insolation (flux of solar irradiation) recalculated as the average flux spread over the whole surface of Io (not just its sun-lit side). Assume that 63 % of incoming solar radiation is scattered and 37 % absorbed.

b) Draw conclusions as to what is heating Io more: tidal interaction with Jupiter or the irradiation by the sun.

c) If its surface cools down according to the Stefan-Boltzmann law, and summing up the tidal and radiative (absorbed) fluxes, what is the expected mean temperature T of Io's surface?

4) **Bond Albedo.** Show that the Bond albedo of a Lambertian surface is $1.5A_0$ (i.e. 1.5 times its geometric albedo).

5) Problem 3.26 (a, b, c, and d) from the **second** edition of *Planetary Sciences* (de Pater & Lissauer).

6) Read Journal Club paper 3 and paper 4, and submit four questions you have about **each** paper.