News and Reminders

Homework 1 is due Monday. Note the addition of Problem 6, which requires you to come up with four questions on JC paper 1.

Next reading quiz: Wednesday, 9/11.

JC papers, dates and presenters:

1) *Evidence for Hidden Nearby Companions to Hot Jupiters* – Sep. 16 - Sarah

2) *Tilting Uranus via Spin-Orbit Resonance with Planet Nine* – Sep. 25 - Lily and Drew

3) *Photochemically produced SO_2 in the atmosphere of WASP-39b* – Oct. 7 – Jake and Andy

4) *Galileo Magnetometer Measurements: A Stronger Case for a Subsurface Ocean at Europa* – Oct. 7 – Sam

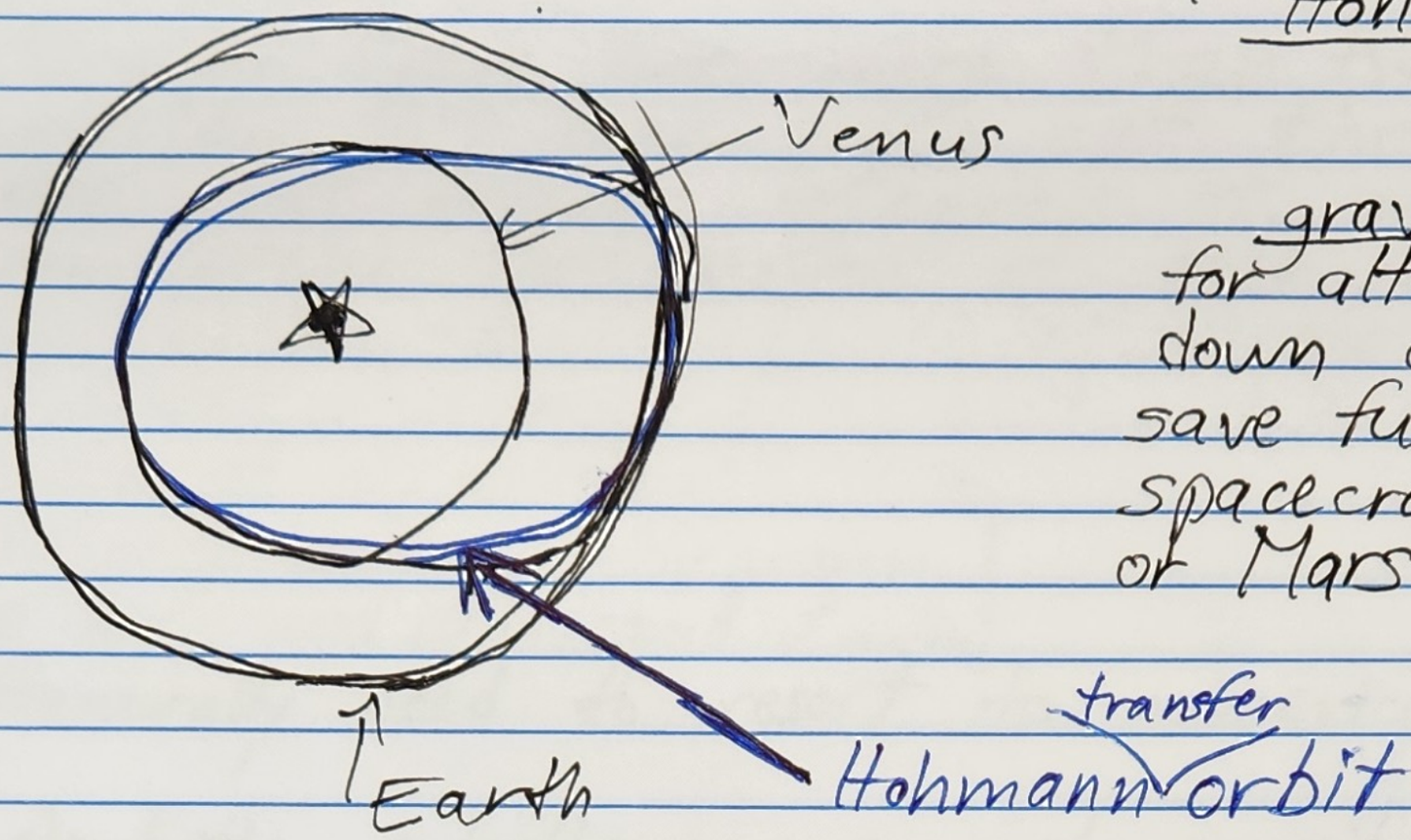
5) *Stable-isotopic anomalies and the accretionary assemblage of the Earth and Mars: a subordinate role for carbonaceous chondrites* – Oct. 28 – Sharleen and Alexis

6) *The geology of Pluto and Charon through the eyes of New Horizons* – Nov. 6 - Ella

7) *Rapid growth of gas-giant cores by pebble accretion* – Nov. 18 - Brett

8) *Halting type I planet migration in non-isothermal disks* – Dec. 2 - Charlie

Spacecraft in the Solar System and Hohmann orbits



gravity assist used for altering orbit (slow down or speed up) and save fuel when sending spacecraft beyond Venus or Mars

e.g. E-V transfer orbit → M-V transfer orbit

use vis viva equation (can be derived from the energy equation) to calculate velocity needed to leave Earth or arrive at target:

$$v^2 = GM\left(\frac{2}{r} - \frac{1}{a}\right)$$

generalized for elliptical orbits, where $r \neq a$

The three-body problem:

Usually there are more than two bodies in a system. But even with just three bodies, there are not enough integrals of motion to specify completely the motion of each body (6x3 dof)

2: $\frac{d^2\vec{r}}{dt^2} = -\frac{GM}{r^2}\hat{r}$ 3: 18 dof ⇒ reduces to 12 ⇒ 6N for N-body problem but $12 > 10$!

(com moves at cst. velocity
⇒ reduces to 4-body problem
6 dof and 10 integrals of motion)

Generally need to resort to numerical integrations.

Analytic solutions are only possible in some limiting cases.

Circular restricted three-body problem:

Third body has negligible mass, circular orbit.

The problem is thus reduced to the study of the motion of only the test particle in the field of the two co-orbiting primaries (because the orbits of m_1 and m_2 conform to the solution of the 2-body problem and are thus known).

Recall: with judicious coordinate change ($\vec{r} = \vec{r}_1 - \vec{r}_2$), the symmetry of the 2-body problem can be used to find an equivalent 1-body problem ⇒ so we go from 12 dimensions to 6 and $6 < 10$ so we can solve!

Need to move to rotating coordinate system with origin @ CoM of the two primaries, with x-axis passing through them and z-axis perpendicular to orbital plane.

Jacobi constant and Jacobi integral

For the circular R3BP, ~~energy~~ total energy and ang. mom. are conserved as usual, but they are not ~~useful as constants~~ of motion because they do not involve the third body whose motion we are trying to understand.

The energy of the third body is time-dependent. Assume total E is 0

$$\frac{1}{2}\frac{d(v^2)}{dt} = -\frac{dU}{dt} \qquad \text{(mass specific energy)}$$

integrate: $\frac{1}{2}v^2 + C_1 = -U + C_2 \qquad C = C_1 + C_2$

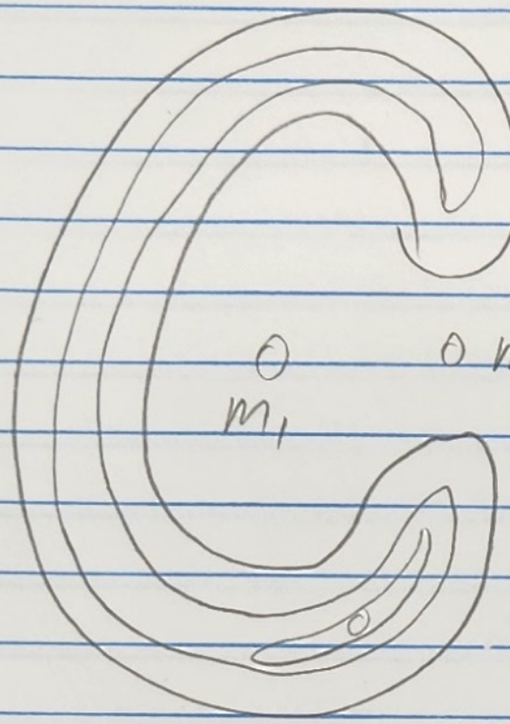
$$\frac{1}{2}v^2 + U = C$$

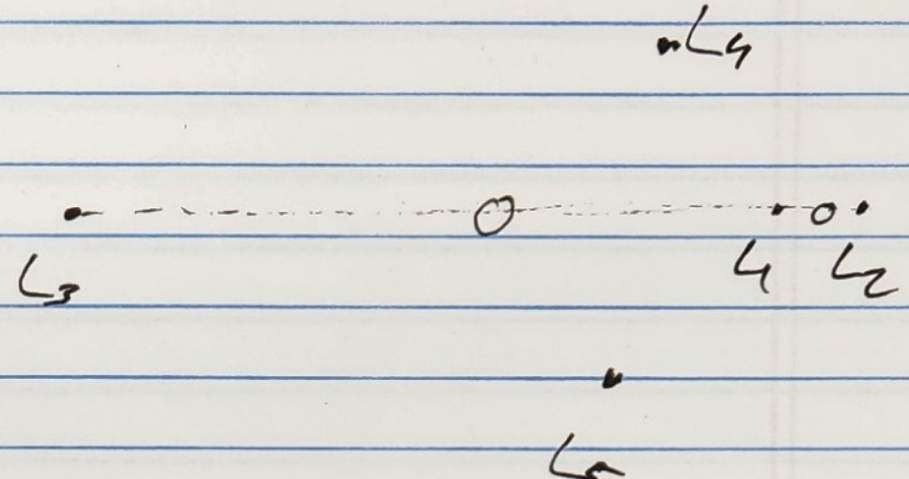
$$\frac{1}{2}v^2 - \frac{1}{2}(x^2+y^2) - \frac{m_2}{\vec{r}-\vec{r}_1} - \frac{m_1}{\vec{r}-\vec{r}_2} = C$$

$\times 2$ (convention)

$$x^2 + y^2 + \frac{2m_2}{\vec{r}-\vec{r}_1} + \frac{2m_1}{\vec{r}-\vec{r}_2} - v^2 = C_J$$

Jacobi integral relates third particle's position and velocity at any point. It is the total energy of the third particle rel. to rotating ref. frame. Not fully solved, but determines forbidden regions.





L_1, L_2 and L_3 are saddle points of the total potential $\Rightarrow$ they are unstable.

L_4 and L_5 together form a 0-velocity curve with smallest CJ. A small perturbation to a particle here causes it to libirate around the point.