

News and Reminders

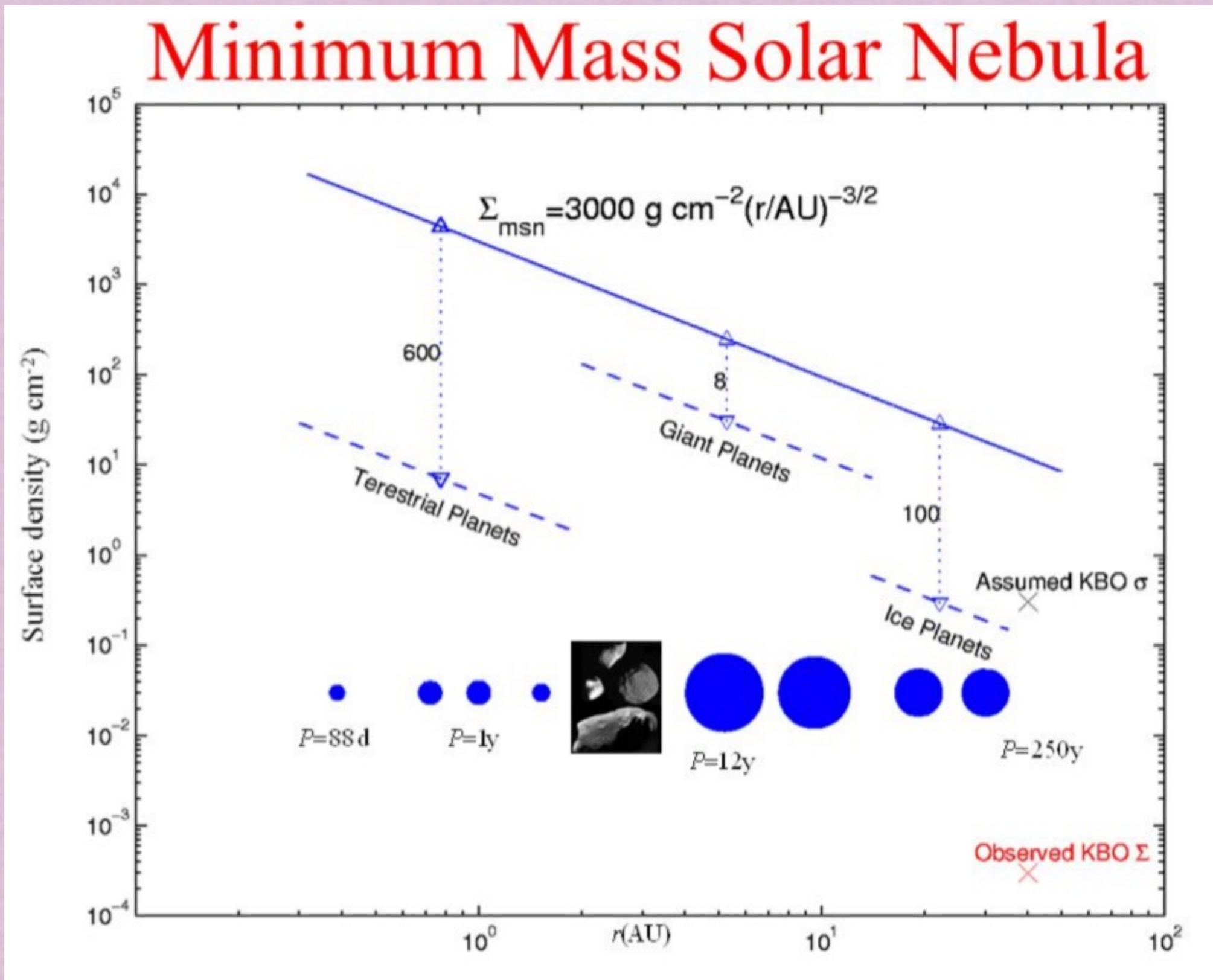
Homework 5 - due Monday, Nov. 18

Wednesday - guest lecture by Dominic Oddo

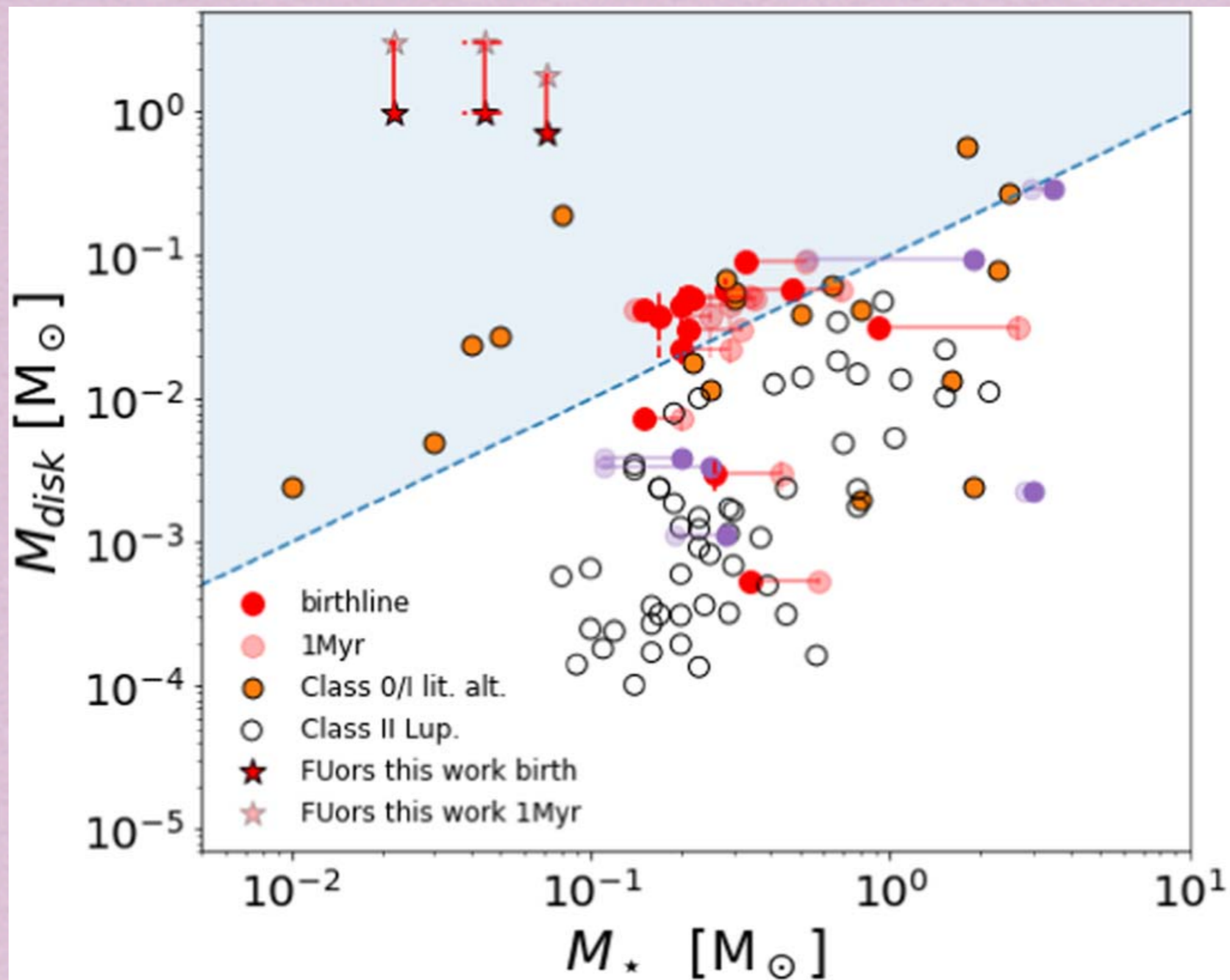
End of semester proposal due dates:

- Abstract due: today
- Proposal due: Monday, Dec. 2

Minimum-Mass Solar Nebula (MMSN)

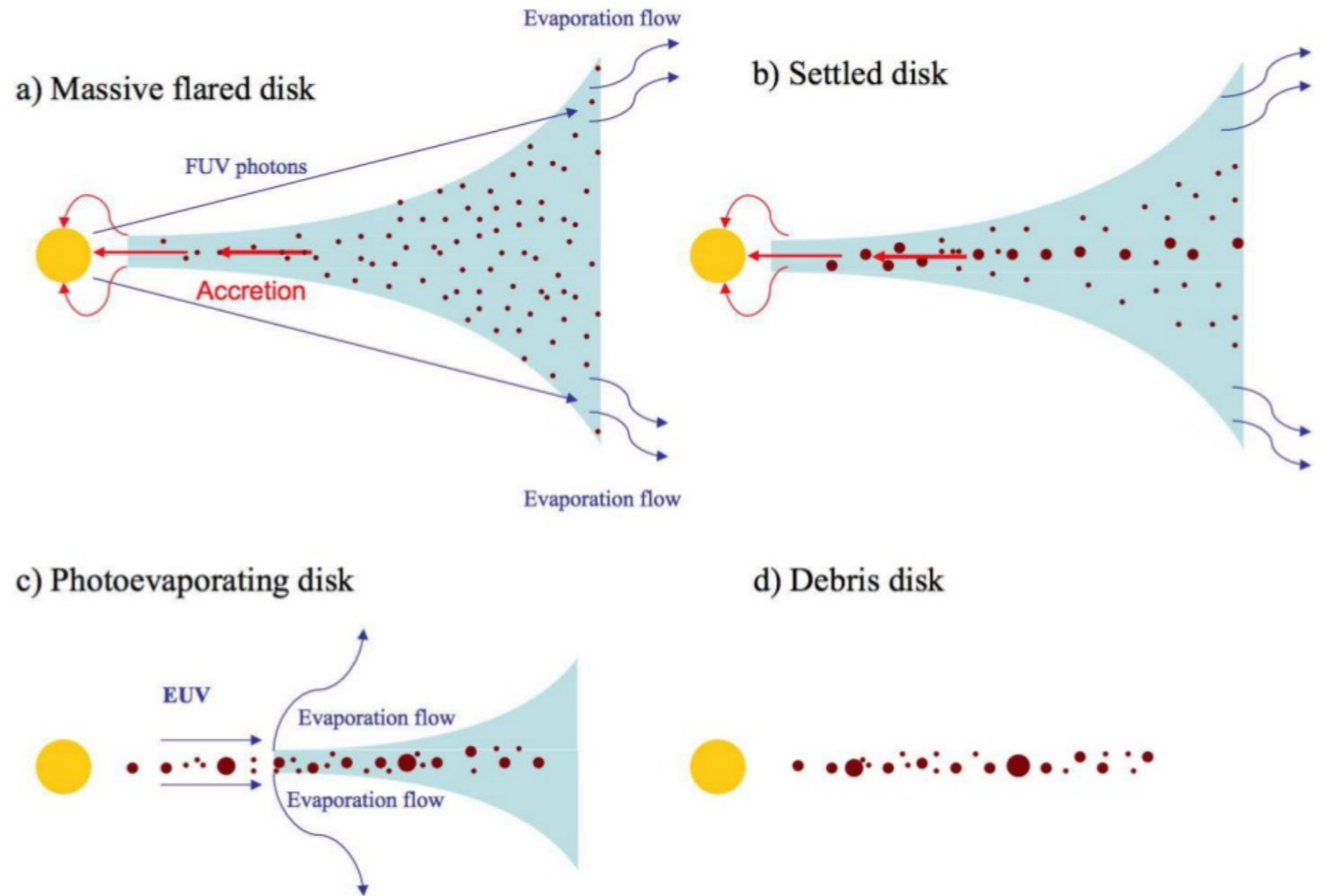


Disk Masses

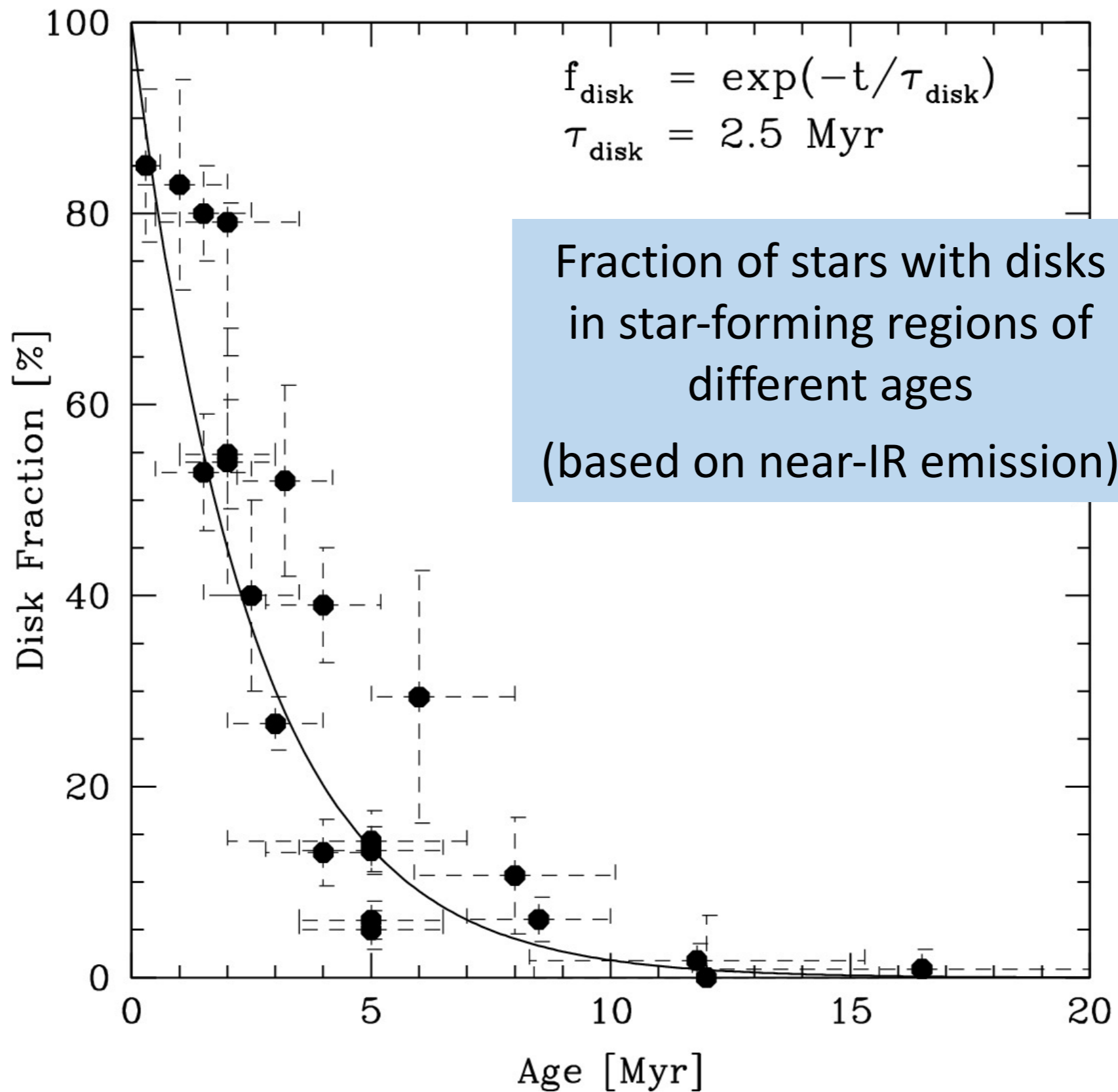


Disk Evolution

Gas is cleared from the disk mainly by a combination of: accretion onto the central star, accretion into gas-giant planets, and photoevaporation



Disk Lifetimes



Planetesimal Formation

It all starts with dust...



Dust grains (<1 micron) are present in atmospheres of giant stars

-> but their formation is still debated

Planetesimal Formation

Krus & Wurm (2018)

At this early stage, motion of grains is coupled to gas

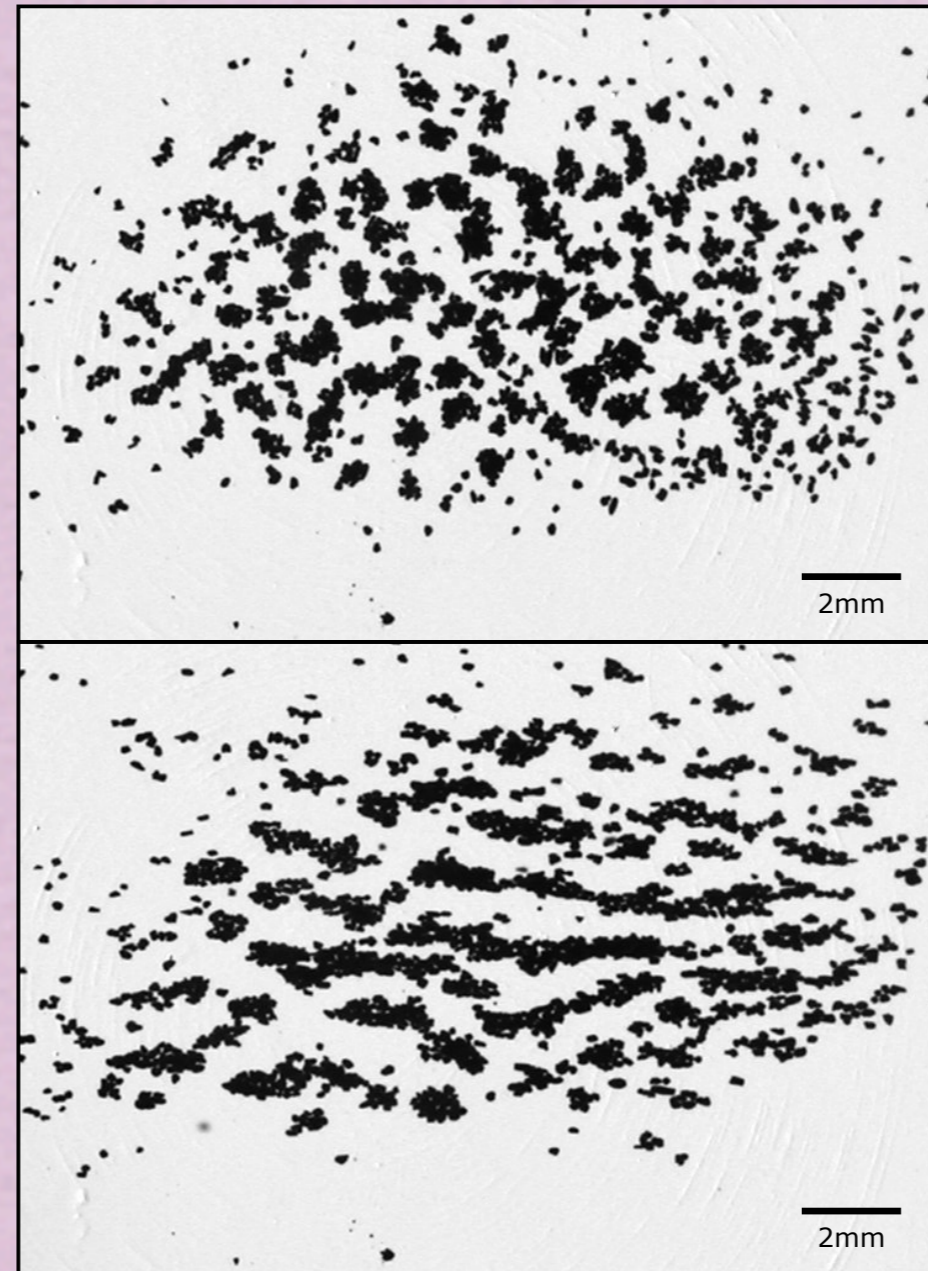
Fractals form, held together by van der Waals force (short-range force from interaction of dipole moments at surface of grains that are in contact)

At ~1 mm: bouncing/fragmentation barrier!

-> but need sticking for growth

-> ongoing research

-> magnetic fields could shift bouncing barrier to larger sizes -> magnetic aggregation



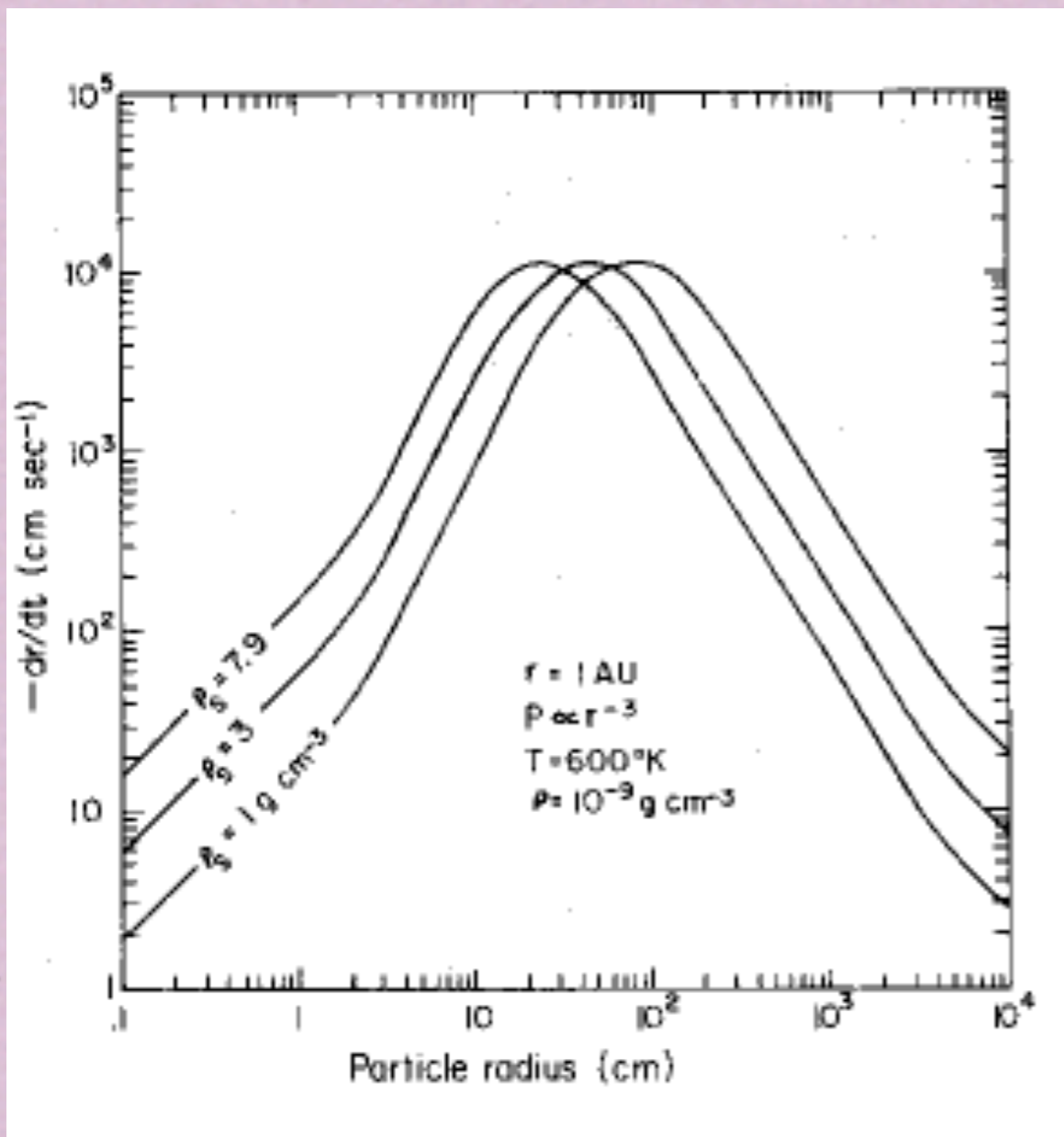
Magnetic field on

Magnetic field off

The Drift Barrier

As particles grow in mass, they decouple from the gas and begin to settle toward the mid-plane of disk

Consider: gas is partially supported against stellar gravity by pressure in radial direction, so gas moves *slower* than Keplerian rate.



Smaller grains are coupled to the gas, but:
Larger particles (mm - cm) move at speeds closer to Keplerian and thus feel a headwind from the slower gas.

-> some angular momentum is removed from particle -> they drift inward

Very large bodies (km-sized) have low surface area to mass ratio, so feel less headwind -> no drift

"Effective gravity" felt by gas:

$$g_{\text{eff}} = \frac{-GM_0}{r^2} - \frac{1}{\rho_g} \frac{dP}{dr} \quad \leftarrow \text{have you seen this before?}$$

centrifugal acceleration

acceleration produced by the pressure gradient

but also: $g_{\text{eff}} = -r\Omega_g^2$ (for a circular orbit)

Angular velocity

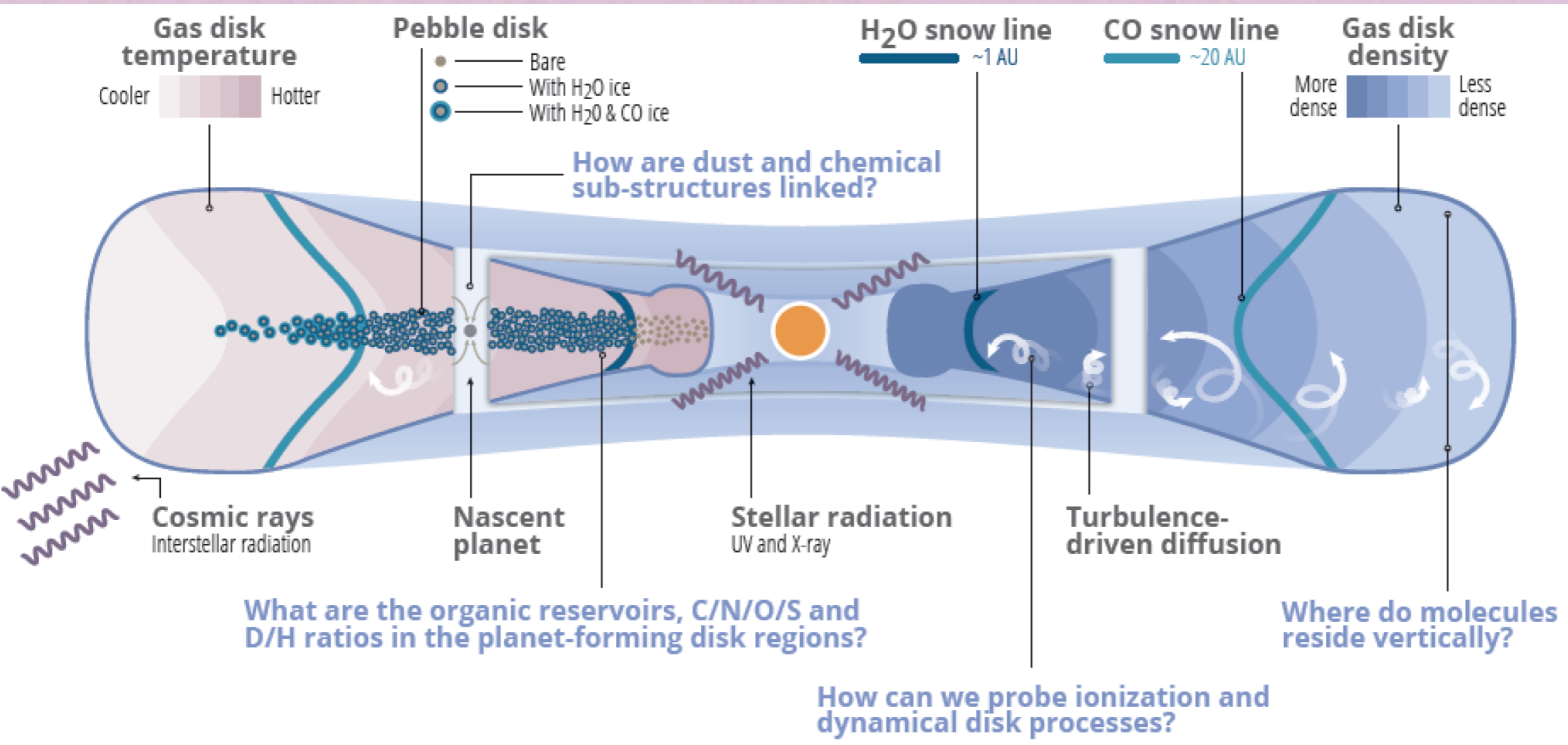
$$\Omega_g = \sqrt{\left(\frac{GM}{r^2} + \frac{1}{\rho_g} \frac{dP}{dr} \right) \frac{1}{r}}$$

$$= \sqrt{\frac{GM}{r^3} \left(1 + \frac{r^2}{GM\rho_g} \frac{dP}{dr} \right)}$$

$$= \sqrt{\frac{GM}{r^3} \left(1 - \underbrace{\left(\frac{-r^2}{2GM\rho_g} \frac{dP}{dr} \right)}_{\sim 5 \times 10^{-3}} \right)}$$

$\sim 5 \times 10^{-3}$

so disk rotates 0.5% slower than Keplerian speed



Planetesimal Formation

Gravitational instability planetesimal formation:

- if dust settles in very thin disk that is also nearly perfectly free of turbulence, then dust disk may fragment into clumps that collapse under own gravity;
- problem: turbulence prohibits these circumstances from being reached.

Streaming instability:

- bodies drift in (from loss of angular momentum), encounter another one and accumulate into a cluster
 - *local* gas is sped up a little by cluster and rotates closer to Keplerian speed
 - headwind on cluster is reduced, and drifts more slowly toward the star
- slower drifting clusters are overtaken and joined by isolated particles from further away, increasing the local density and further reducing radial drift
- -> exponential growth of the clusters

From Planetesimals to Planetary Embryos

Lots of planetesimals floating around.

These $O(1 \text{ km})$ -sized bodies feel much less headwind from the gas.

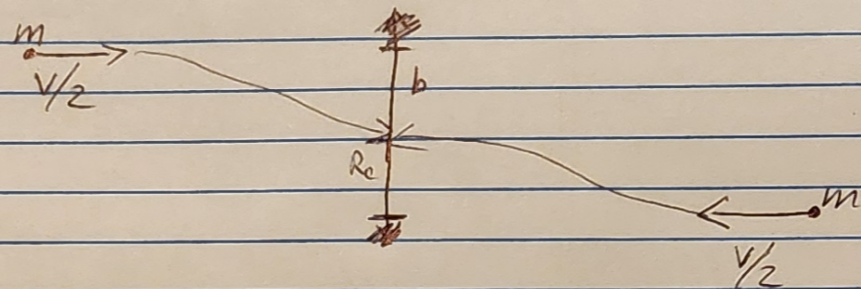
Collisions abound:

- can be mostly inelastic \rightarrow accretion
- elastic \rightarrow fragmentation
- elastic \rightarrow rebound
 - “semi”-Keplerian orbits are changed to random motions



From planetesimals to planetary embryos:

b : impact parameter
(distance of closest approach
without gravity)



Collisions + accretion

relative velocity of each body at infinity
is $v/2$

at closest approach, they have velocity
 v_{max} and separation R_c .

Energy conservation:

$$\frac{1}{2} m \left(\frac{v}{2}\right)^2 + \frac{1}{2} m \left(\frac{v}{2}\right)^2 = \frac{1}{2} m v^2 \times 2 = \frac{1}{2} m v^2$$

$$= \frac{1}{2} m v_{max}^2 \times 2 - \frac{G m m}{R_c}$$

$$= m v_{max}^2 - \frac{G m m}{R_c}$$

Conservation of angular momentum:

$$-m \frac{v}{2} r + m \frac{v}{2} (r+b) = 2 m v_{max} \frac{R_c}{2} \quad \left(\text{since no radial component of velocity at point of closest approach}\right)$$

$$\frac{v b}{2} = v_{max} R_c$$

$$\Rightarrow v_{max} = \frac{v b}{2 R_c}$$

(since no radial component of velocity at point of closest approach)

Let $R_s =$ sum of radii of the two bodies

if $R_c < R_s \Rightarrow$ collision

if $R_c > R_s \Rightarrow$ flyby

$$b^2 = \frac{4 v_{\max}^2 R_c^2}{v^2} = \frac{4 R_c^2}{v^2} \left(\frac{1}{4} m v^2 + \frac{G m^2}{R_c} \right) \frac{1}{m}$$
$$= R_c^2 + \frac{4 R_c G m}{v^2}$$

So the largest value of b that gives a collision is:

$$b = \sqrt{R_s^2 + \frac{4 R_s G m}{v^2}}$$

if b is larger than this, it means that $R_c > R_s$

can write as function of v_{esc} ($v_{\text{esc}} = \sqrt{\frac{4 G m}{R_s}}$)

$$b = R_s \sqrt{\left(1 + \frac{v_{\text{esc}}^2}{v^2} \right)}$$

Can define a gravitational focusing factor;

$F_g = 1 + \left(\frac{v_{\text{esc}}}{v} \right)^2$ and a cross-section for collisions

$$\Pi = \pi R_s^2 \left(1 + \left(\frac{v_{\text{esc}}}{v} \right)^2 \right) = \pi R_s^2 F_g$$

When $v \ll v_{\text{esc}}$, growth is much faster due to gravitational focusing

Gravitational Focusing

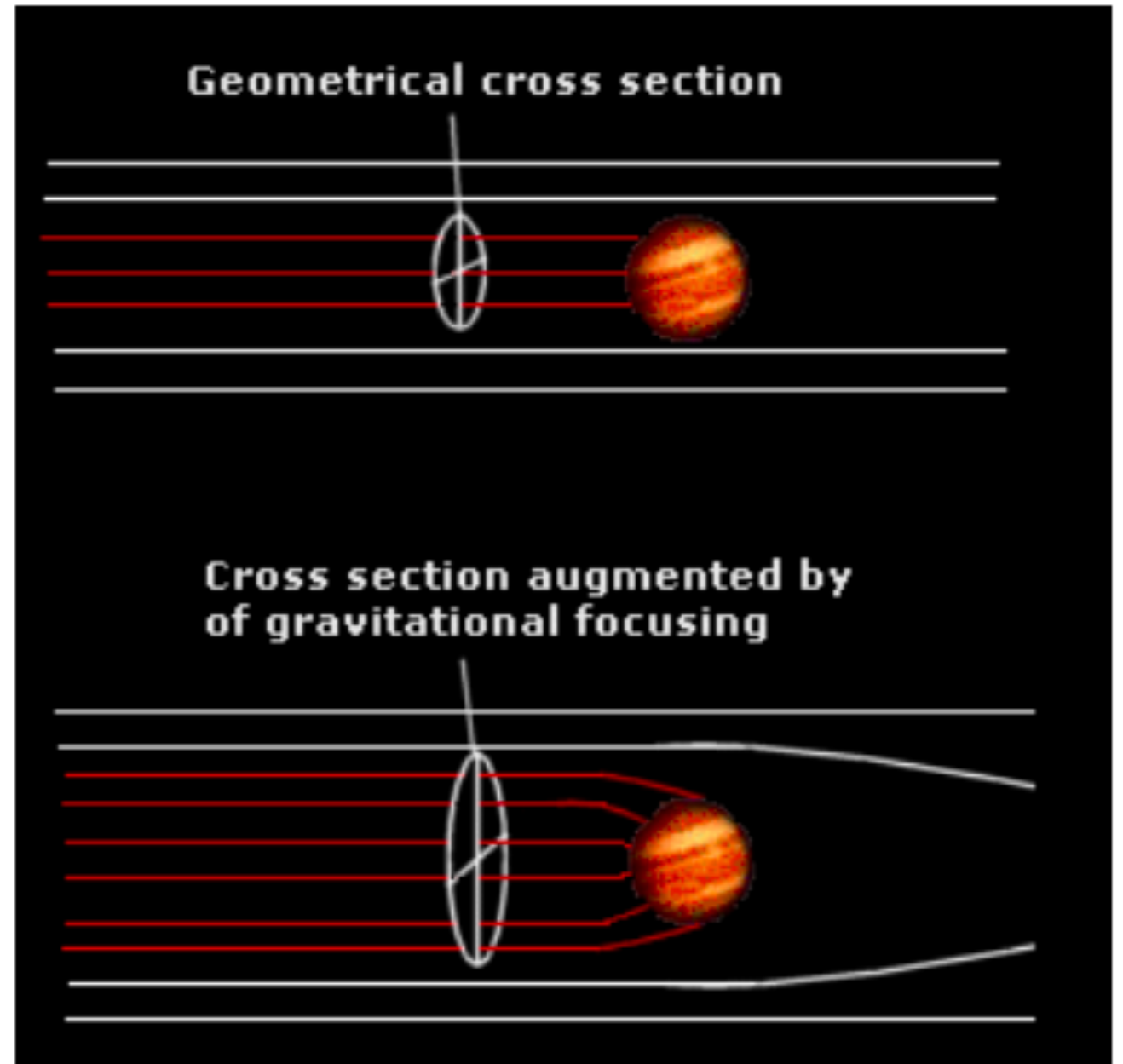


Without gravitational focusing:

$$\Gamma = \pi R_p^2$$

With gravitational focusing:

$$\Gamma = \pi b^2 = \pi (R_c^2 + 4R_c Gm/v^2)$$



Growth rate:

$$\frac{dM}{dt} = \rho_{sw} v_{rms} \Gamma$$

where ρ_s is the density of the planetesimal swarm and v_{rms} is the dispersion velocity

$$= \rho_{sw} v_{rms} \pi R_s^2 F_g \propto M^{2/3}$$